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An Analysis of Errors Made by Grade 7 Students in Solving Simple Linear Equations in One Variable

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Abstract— This study was carried out to determine the errors made by Grade 7 students in solving simple linear equations in one variable. The participants of the study were the eighty-five (85) Grade 7 students of Moonwalk National High School officially enrolled for the School Year 2018-2019. To gather data, the researcher developed a six-item open-ended test involving solving simple linear equations in one variable. The development of the test was primarily based from the literature. Likewise, the errors found were categorized according to the types of errors in literature. Frequency, percentage and rank were used to analyze the data gathered. Results show that a total of 309 errors were made by the Grade 7 students. These errors are categorized in the literature as Switching Addends Error, Transposing Error, Omission Error, Number Line Error, Division Error, Absence of Structure Error, Inability to Isolate Variable Error, and the Misuse of Additive Inverse Error. Results further show that the Number Line error was the most frequent error made by the Grade 7 students in solving simple linear equations in one variable followed by the Misuse of Additive Inverse Error. However, the least frequent error was the Inability to Isolate Variable Error. The study recommends that when teachers teach solving linear equations in one variable, they should be aware of these errors and other errors that may be made by the students. Moreover, they should design techniques and strategies that would help the students overcome these errors.

Index Terms— error analysis, solving simple linear equations, linear equations in one variable, number line error, misuse of additive inverse error, switching addends error, transposing error, omission error, division error, absence of structure error, inability to isolate variable error

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1 INTRODUCTION

To teach mathematics is challenging especially to those students who have poor foundation in the subject. Similarly, to learn in math is neither a piece of cake nor a simple 1, 2, 3's for if not most, some of the students. As a result, studies to help students improve their mathematical skills have been prevalent. These undertakings, among others, focused on the effectiveness of some teaching styles, new techniques and methods in improving the achievement of the students in mathematics. However, only a few tried to analyze errors made by the students while solving equations, inequalities and problems in mathematics. Determining students' errors is a very good technique to provide students with instruction targeting their area of needs. Lai (2012) defines error analysis as a method commonly used to identify the cause of student errors when they make consistent mistakes. It is a process of reviewing a student's work and then looking for patterns of misunderstanding. Errors in mathematics can be factual, procedural, or conceptual, and may occur for a number of reasons. Riccomini (2016) argued that as math problems become more complex, students need to go through a series of steps to solve problems. An error in any of these steps can cause failure in the final response. As a result, it is important to identify errors, especially error patterns, and provide targeted instruction to correct the error. In line with this, the researcher became interested to determine the errors made by the grade 7 students in solving simple linear equations in one variable. Mathematics is very important and is widely used in daily life. Thus, it is reasonable to help students address their difficulties in learning maths through identifying the errors they frequently make and provide them instruction utilizing techniques that would overcome these errors.

1.1 Problem Statement

How many errors do the students make in solving simple linear equations in one variable? What are these errors? How are these errors categorized according to the literature? How frequent do the students make these errors?

2 METHODOLOGY

2.1 Research Design

This study is quantitative in nature and utilizes the descriptive research design.

2.2 Sampling

The participants of this study were the eighty-five (85) Grade 7 students of Moonwalk National High School officially enrolled for the School Year 2018-2019. These students are the researcher's students in his mathematics class who came from sections Kepler and Huygens. Moreover, the participants' ages range from 11 to 15 years old while 40 are males and 45 are females. All of these participants are Filipino seventh grade students. The participants were purposively selected by the researcher.

2.3 Instrument used

The researcher developed a 6-item open-ended test involving solving simple linear equations in one variable. The development of the 6 linear equations in the test was primarily based from the study of Hall (2002) when he analyzed the linear equation errors made by the first year secondary students. The following are the 6 linear equations developed based from the large-scale study of Hall.

Equation 1: $8x = 72$
Equation 2: $x + 5 = 3$
Equation 3: $\frac{16}{x} = 2$*:*
Equation 4: $6x - 7 = 23$
Equation 5: $4x - 3 = x - 2$
Equation 6: $10 + \frac{x}{2} = 2$

2.4 Data Analysis

The solutions of the participants for each equation were carefully examined by the researcher to look for errors identified in the literature by the study of Hall (2002), Carry, Lewis, and Bernard (1980), Matz (1981), and Kieran (1984 & 1992). The data gathered were analyzed using frequency, percentage and rank.

3 RESULTS

Table 1 shows that a total of 452 attempts were made by the participants to solve the linear equations however, only 109 or that is only almost a quarter arrived at the correct answers. In line with this, it can be stated that the participants encountered errors while doing their solutions since many attempted while only a few got the correct answers. Solving equations is a particularly important concept in algebra and one that causes confusion for students (Cai & Moyer, 2008)

Table 1

Frequency of Attempts and Correct Answers to each Linear Equation

Equation	***Attempt***	***Correct***
1. $\mathbf{8x = 72}$	80	65
2. $\mathbf{x + 5 = 3}$	75	5
3. $\mathbf{\frac{16}{x} = 2}$	73	21
4. $\mathbf{6x - 7 = 23}$	81	19
5. $\mathbf{4x - 3 = x - 2}$	77	3
6. $\mathbf{10 + \frac{x}{2} = 2}$	66	2
Total	**452**	**109**

The solutions of the participants for each linear equation in the test were carefully examined to determine errors. Results in Table 2 revealed that a total of 309 errors were made by the participants in solving simple linear equations in one variable. These errors were found to be similar to those of the errors categorized by Hall (2002) in his large scale study. Among these errors are the switching addends error (*f=15 or 4.85%*), transposing error (*f=43 or 13.92%*), omission error (*f=45 or 14.56%*), number line error (*f=87 or 28.16%*), division error (*f=18 or 5.83%*), absence of structure error (*f=42 or 13.59%*), inability to isolate variable error (*f=9 or 2.91%*), and the misuse of additive inverse error (*f=50 or 16.18%*). Solving linear equations is really difficult and confusing especially for students who have poor foundation in mathematics and understanding of the concepts and procedures in algebra. As a result, a number of errors may they make in trying to solve the equations. Similarly, research studies (Herscovics & Linchevski, 1994; Kiera, 1997; Boulton-Lewis & et al., 2000; Radford, 2000; Hall, 2002a; Vlassis, 2002) had identified a number of errors and misconceptions on students' understanding of algebra.

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Table 2

Frequency of Errors Made in Solving Simple Linear Equations in One Variable

Error	***f***	**%**
Switching Addends Error	15	4.85
Transposing Error	43	13.92
Omission Error	45	14.56
Number Line Error	87	28.16
Division Error	18	5.83
Absence of Structure Error	42	13.59
Inability to Isolate Variable Error	9	2.91
Misuse of Additive Inverse Error	50	16.18
Total	**309**	**100.00**

The errors were ranked according to the frequency it was made by the participants. Table 3 reflects that the number line error was the most frequent error made. This was followed by the misuse of additive inverse error, omission error, transposing error, absence of structure error, division error, switching addends error, and then the least frequent was the inability to isolate variable error. On the other hand, these results are contrary to that of Hall (2002) for he found that the most frequent errors as a whole were the Transposing, Division, and Switching Addends errors. This difference may be attributed to the fact that the participants did not even master the skills in operating with integers. Interestingly, the result corroborated the strong belief that students will encounter a number of difficulties in learning higher mathematics without mastering the prerequisite skills.

Table 3

Rank of Errors Made in Solving Simple Linear Equations in One Variable

Error	**Rank**
Number Line Error	1
Misuse of Additive Inverse Error	2
Omission Error	3
Transposing Error	4
Absence of Structure Error	5
Division Error	6
Switching Addends Error	7
Inability to Isolate Variable Error	8

4 CONCLUSION

4.1 Summary of Findings and Conclusion

The study has found that the participants have made a number of errors in solving simple linear equations in one variable. This may show that the participants did not fully understand the lesson when taught by the researcher to them. Since the study was conducted after they were taught with the lesson on solving linear equations in one variable. Meanwhile, the number line error was the most frequent error made in the participants' attempt to solve the linear equations. This was followed by the misuse of additive inverse error, omission error, transposing error, absence of structure error, division error, switching addends error while the least frequent was the inability to isolate variable error. These findings may lead the conclusion which may state that the participants really lack conceptual and procedural knowledge and understanding in algebra, particularly in solving linear equations in algebra.

4.2 Discussion

The descriptive statistics shows that the participants exerted efforts to solve the linear equations. On the other hand, most of their attempts failed to arrive at the correct answers. This suggests that errors occurred while the participants were solving the equations. And it was found that the most frequent error was the number line error. According to Hall (2002), one of the subordinate skills in learning to solve simple linear equations is the ability to simplify expressions such as -3 + 1, which may imply about operations with integers. In this study, the number line error was seen in all equations in the test but mostly in equations 2 and 5. Figure 1 presents an example of the number line error made by a participant. It can be observed that the participant's error was when he answered 2 for 3-5 which should be -2.

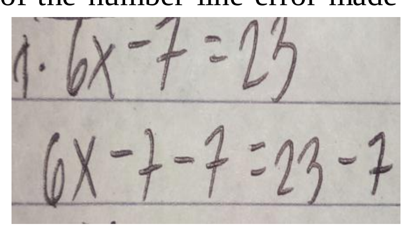

Figure 1

However, the second most frequent error made, the misuse of additive inverse error was mostly

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observed in equations 3 and 4. According to Hall (2002), this error may show some understanding of the balance analogy, in that the pupil has done the same to both sides. Only that the participant incorrectly determined the opposite of a number (additive inverse) that he/she would add/subtract to both sides of the equation. Just like the work shown in Figure 2, the participant subtracted 7 to both sides of the equation instead of adding 7 which should be the right procedure in order for him to arrive at the correct answer. The omission error,

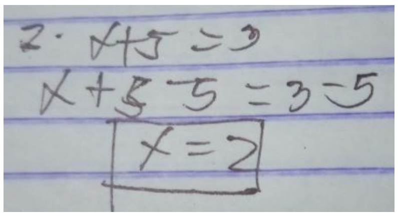

Figure 2

according to Hall (2002), usually occurs in the middle of the solution of the problem, and occurs while a pupil is using a structural method. This happens when a pupil omits letter/s or number/s in his solution without any reason. In the present study, this error usually happened in equation 5. Figure 3 shows that the participant just omitted x for no reason after she employed the property of equality with the correct use of additive inverse. Probably because she could

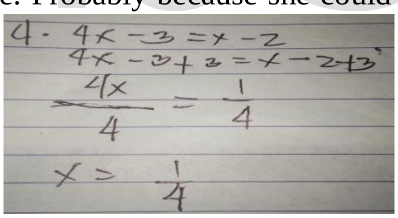

Figure 3

not handle the problem anymore, she just dropped x for no reason to simplify the equation. Meanwhile, the transposing error was frequently observed in equations 3 and 6 in this study. Figure 4 shows a case of this error when the participant just

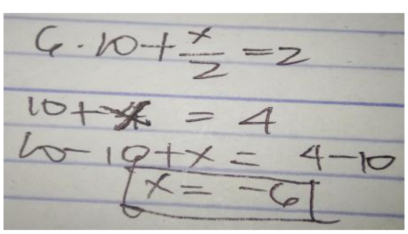

Figure 4

multiplied the denominator and the constant at the right side of the equation yielding 10 + x = 4. Transposing error may spring from a pupil constructing a method in accordance with what appears to work often (Hall, 2002). On the other hand, the absence of structure error as defined by Hall (2002) refers to the inability of the participant to do to the other side of the equation what he did on one side. The cases of Absence of Structure error were those in which the pupil demonstrated a lack of understanding of 'doing the same to both sides'. In the study, this error was frequently observed in equation 5. As seen in Figure 5, the participant subtracted 3 at the left side but did not do it at the right side. This shows that the participant may lack understanding of the property of

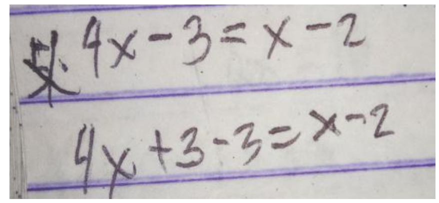

Figure 5

equality. However, the division error was frequently observed in equation 1. The Division error may seem a relatively unimportant one in the context of solving linear equations. However, until such division is mastered, pupils without calculators may often be unable to find non-integer solutions to linear equations (Hall, 2002). Figure 6 shows that

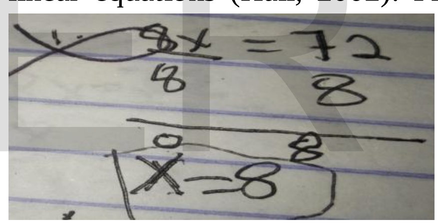

Figure 6

the error made by the participant was when he answered 8 as the quotient of 72 and 8. The second to the least frequent error was the switching addends error. The Switching Addends error, where x + 37 = 150 is judged to have the same solution x = 37 + 150 (Kieran, 1992, page 402). In this study, this was observed most frequently in equation 2. Figure 7 shows that 5 was transferred to the right side to become an addend of 3 giving the result of x=8. Finally, the least frequent error was the inability to isolate variable error. According to Hall (2002) this error may arise because the pupil does not realize what must be done i.e. even towards the end of the question, we must do the same to both sides. It was frequently

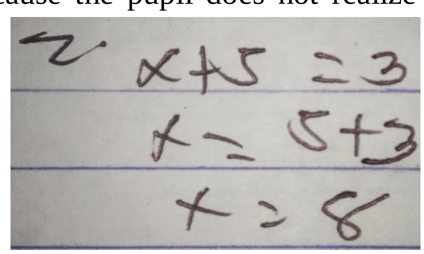

Figure 7

observed in equation 3. Figure 8 shows that the participant did not know what to do next after arriving at 2x=16 by cross-multiplication process.

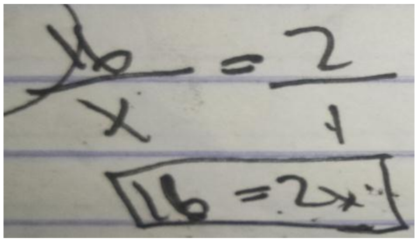

Figure 8

These results suggest that a great challenge awaits to the teacher in teaching students how to solve simple linear equations in one variable.

4.3 Limitations of the Study

The first limitation of the study is that it analyzes errors of participants based from the errors available in the literature. Likewise, the number of participants is small to represent the entire population. Moreover, the errors that may be made were only limited to simple linear equations which are based on the literature. This study obtained results from self-report tests, thus there is no guarantee that the students answered honestly. In addition, the results of the present study is limited only to the responses of the participants, thus it cannot be generalized to the entire population.

4.4 Recommendations for Future Study

To ascertain the results of this study, a replication of this study should be done. Future studies should also be done with larger sample size and other ways of data gathering such as interviews and FGDs. Moreover, future studies should not only focus on simple linear equations. Researchers should also analyze errors made in solving more complex linear equations to determine more errors and understand the thinking of the students in solving linear equations.

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